Coordination and Multiplicity:
Multiple Equilibria and Self-Fulfilling Crises

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Calvo (1988)

Servicing the Public Debt: The Role of Expectations

"As one looks around the world one is impressed by the wide variety of languages, cultures, and even perceptions of the same phenomena. However, the topic of the government budget is one of the few where the range of issues and degree of emotions appears to find a common denominator. The government budget lies at the basis of political campaigns and plays a significant role in the evaluation of government policy."

- Model of debt repudiation via inflation or default
- Multiple equilibria are possible
- Government policy suffers from time inconsistency

Debt repudiation (real model)

- Two periods, $t = 0, 1$
- Unit measure of consumers and a government
- Government budgets

$$g_0 = b$$
$$\tau = (1 - \theta) bR + g_1 + \alpha \theta bR$$

  - θ = proportion of defaulted debt, α resource cost of repudiation
  - g_0, g_1 exogenous and > 0
- Households' budgets

$$y_0 = k + b$$
$$kR^* + bR(1 - \theta) = c + \tau + z(\tau)$$

  - no-arbitrage condition: $R^* = R(1 - \theta)$
  - $z(\tau)$ tax distortion, increasing and convex

Government problem

Maximise household consumption by minimising default + tax distortions

$$\max_{\theta\in[0,1]} c = \min_{\theta\in[0,1]} \alpha\theta bR + z\left(\underbrace{bR(1-\theta(1-\alpha))+g_1}_{\tau}\right)$$

First best: solve problem at $t=0$ with commitment. Plug no-arb into problem, get $\theta^{FB}=0$

Second best: FOC when θ interior: $z'(\tau)=\frac{\alpha}{1-\alpha} \quad \rightarrow \tau^*$

θ interior iff $\tau \in [g_1+\alpha bR, g_1+bR]$

Plugging the no-arbitrage condition into the expression for taxes

$$\tau = g_1+\alpha bR+(1-\alpha)bR^*$$

Equilibria

Eqm #1: $R = R^*$, $\tau < \tau^*$, $\theta = 0$

Eqm #2: $R > R^*$, $\tau = \tau^*$, $\theta \in (0, 1)$

Lower τ^* means Eqm #2 closer to R^*. If $\tau^* < g_1 + bR^*$, no eqm with $b > 0$ exists

Interpretation:

- expected default is high when interest rates are high
- expected default is low when interest rates are low

Equilibrium multiplicity arises from investors' failure to coordinate

Note: τ^* (eqm tax distortion) increasing in repudiation cost α (default distortion)

- α low enough means no eqm with $b > 0$: default temptation too high, no lending in the first place
- higher α ensures existence, but makes Eqm #2 worse
- Eqm #2 is Pareto-dominated by Eqm #1

Commitment and time consistency

Commitment increases welfare

- In both frameworks, default has deadweight loss
- Optimal thing to do is commit to full repayment, then $R = R^*$ and select good equilibrium

Time consistency

- Suppose the govt "promises" full repayment, but investors deviate and bid $R^*/(1-h)$
- Then it's not optimal for govt to keep promise and consume $c_R = y - bR^*/(1-h) < c_D$
  $\Rightarrow$ time inconsistency
- If investors anticipate deviation, then bad eqm exists
- If govt can truly commit, then deviation would be feasible but is off the equilibrium path

Self-fulfilling inflation

- Inflation is the same as debt repudiation
- Let the real return on bonds be given by $R\frac{P_0}{P_1}$
- Then $1-\theta = \frac{P_0}{P_1}$ and no-arbitrage implies $R^* = R(1-\theta)$
- Paper assume a simple money demand function and seigniorage, not key for results
- Assume increasing and convex inflation cost $\mathcal{R}(\theta)$. Government problem becomes

$$\max_{\theta\leq 1} c = \min_{\theta \leq 1} \mathcal{R}(\theta) + z\left(\underbrace{bR(1-\theta)+g_1}_{\tau}\right)$$

Multiple equilibria

- Assume increasing and convex inflation cost $\mathcal{R}(\theta) = \beta\frac{\theta^2}{2}$. Government problem becomes

$$\max_{\theta \leq 1} c = \min_{\theta \leq 1} \mathcal{R}(\theta) + z\left(bR(1-\theta) + g_1\right)$$

- **First best**: solve from $t = 0$ with commitment, plug in no-arb, get $\theta^{FB} \perp R$

$$\mathcal{R}'(\theta) = 0$$

- **Second best**: solve from $t = 1$ without commitment, get $\theta^{SB} \uparrow R$ and $\theta^{SB} > \theta^{FB}$

$$\mathcal{R}'(\theta) = bRz'(\tau)$$

$$\beta\theta = b\frac{R^*}{1-\theta}z'(bR^* + g_1)$$

$$\theta \leftrightarrow R!$$

Takeaways

- Expectations play a key role in many dimensions of macro, including govt policy
- Inflation and default policies are prone to time inconsistency
- Without commitment ($\approx$ credibility), expectations may be self-fulfilling
- Solutions?
  - Case 1: debt rationing if R too high, or asset purchases that cap R
  - Case 2: debt indexation, i.e. borrowing in foreign currency

A different framework

Assume *external* debt, and $b = g_0$

$$\begin{aligned} c_R &= y - bR \\ c_D &= y(1-z) - bR(1-h) \end{aligned}$$

Repay iff

$$c_R \geq c_D \quad \Leftrightarrow \quad bR \leq \frac{z}{h} =: x$$

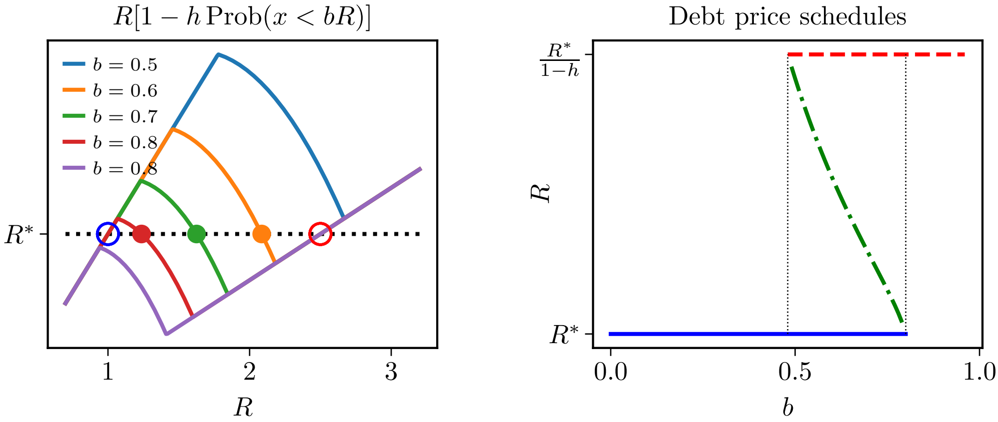
Assume $x \sim F[\underline{x}, \overline{x}]$ and $\overline{x}(1-h) < \underline{x}$. Risk neutral investors no-arbitrage condition

$$R[1 - h\,\text{Prob}(x < bR)] = R^*$$

Equilibria

- Risk-free debt: $R = R^*$, $bR^* \leq \underline{x}$, $c = c_R = y - bR^*$
- Sure default: $R = \frac{R^*}{1-h}$, $b\frac{R^*}{1-h} > \overline{x}$, $c = c_D = y(1-z) - bR^*$
- by continuity, $\exists$ 3rd unstable equilibrium

Equilibria



$R[1 - h\,\mathrm{Prob}(x < bR)]$
$b = 0.5$
$b = 0.6$
$b = 0.7$
$b = 0.8$
$b = 0.8$
R^*
1
2
3
R
Debt price schedules
$\frac{R^*}{1-h}$
R
R^*
0.0
0.5
1.0
b

Bassetto (2005)

Equilibrium and Government Commitment

- We have seen that the government ability to commit has important implications
- It can help avoid multiple equilibria, and "select" the best possible (Ramsey) equilibrium
- This paper highlights possible issues in such selection
- In general, applies to problems where large player (govt) interacts with continuum of small players (e.g. households, investors, etc)

Optimal taxation example

- Two periods, govt and continuum of households
- Households budget constraints

$$\omega \geq c_0 + k$$
$$c_1 = Rk(1 - \tau)$$

- Government budget constraint at $t = 1$

$$G = \tau Rk$$

- Household preferences $U(c_0, c_1, G)$
- Govt preferences $U(C_0, C_1, G)$
- U strictly concave, increasing

Useful concepts

Competitive equilibrium:

- a symmetric allocation k (the rest is implied)
- a government policy (τ, G)

such that

- households maximise utility
- $G = \tau R k$

Ramsey equilibrium: the *best* CE (τ^r, G^r) (and k^r) for the government

Key, non-trivial question: how is it implemented? Can govt make sure

- Govt moves first, announces (τ^r, G^r)
- Households choose k taking govt policy as given

With this *Ramsey* timing, (τ^r, G^r) is unique equilibrium $\Rightarrow$ households best-respond to Ramsey policy (choose k^r)

Off equilibrium paths

But what if households set $k = 0$? Then

$$\tau^r RK = 0 < G^r$$

This *can* be an equilibrium since $G^r = 0$ might imply no incentive to save

So what should households believe about G, or about what everyone else is doing?

Schelling timing

Alternative: focus on feasibility of govt actions, respecting timing

Govt should commit to *strategies* rather than *actions*: $(\tau, G) : [0, \omega] \to [0, 1] \times \mathbb{R}_+$ subject to

$$\tau(K)RK \geq G(K) \quad \forall K \in [0, \omega]$$

Schelling (1960):

> *The threat differs from the ordinary commitment, however, in that it makes one's course of action conditional on what the other player does. While the commitment fixes one's course of action, the threat fixes a course of reaction, of response to the other player. The commitment is a means of gaining first move in a game in which first move carries an advantage; the threat is a commitment to a strategy for second move.*

Schelling implementability

Govt may or may not be able to ensure Ramsey outcome is uniquely implementable

Assume $U(v(c_0, c_1), G)$ with $U_1 > 0$ for any G

- G does not affect MRS between c_0, c_1
- Govt chooses τ^r such that $K = K^r$, lets G adjust
- $k = 0$ not an eqm, Ramsey outcome is Schelling-implementable

Assume $u(c_1, Gc_2)$

- No matter what the govt does, $K = G = 0$ is an eqm
- Schelling timing reaches different outcome (multiple eqa!) than Ramsey: govt cannot uniquely select the eqm it desires

Remarks

- In the 2nd example, adherence to the Ramsey policy is simply not feasible
- Ramsey timing is what most of the macro literature assumes!
- But in most macro applications, the govt commits to strategies to satisfy its budget constraint

Sovereign debt application

Typically models of sovereign debt or FX crises feature lack of commitment

Here, an example where the problem is not lack of commitment but lack of options

Continuum of households

- Domestic and foreign consumption c_h, c_f and hours worked $n \in [0, \bar{n}]$
- Preferences $U(c_h, c_f, n)$
- Labour n produces output $n + q$ where $q \sim g(q)$

Government

- Owes b units of debt to foreign creditors, collects labour taxes
- Can default at cost of interrupting foreign trade ($c_f = 0$)

Timing

- Govt commits to strategy
- Households choose labour supply
- Shock q realises
- Government collects taxes and decides debt repayment rate

$$\tau(N+q) \geq b\delta$$

- Households consume home and foreign (if $\delta \geq 1$) goods subject to

$$c_h + c_f \mathbb{1}_{[\delta \geq 1]} = (1-\tau)(n+q)$$

FOCs

$$U_h = U_f$$
$$U_n = \mathbb{E}[U_h(1-\tau)]$$

Households best response

Assume

$$U(c_h, c_f, n) = \frac{a_h c_h^{1-\rho} + a_f c_f^{1-\rho}}{1-\rho} - n$$

Combining households FOCs we get

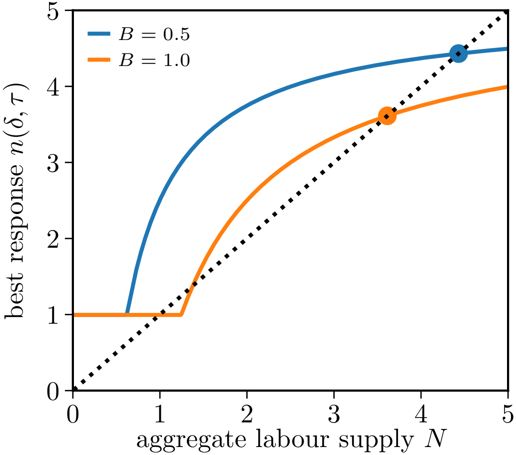
$$n(\delta, \tau) \quad \text{defined by} \quad \mathbb{E}\left[(a_1^{1/\rho} + a_2^{1/\rho}\mathbb{1}_{[\delta\geq 1]})^{\rho}(n+q)^{-\rho}(1-\tau)^{1-\rho}\right] = 1$$

Ramsey outcome $= \max_{\delta,\tau} U(c_h, c_f, n)$ s.t. $n(\delta, \tau)$ and $\tau(N+q) = B\delta$

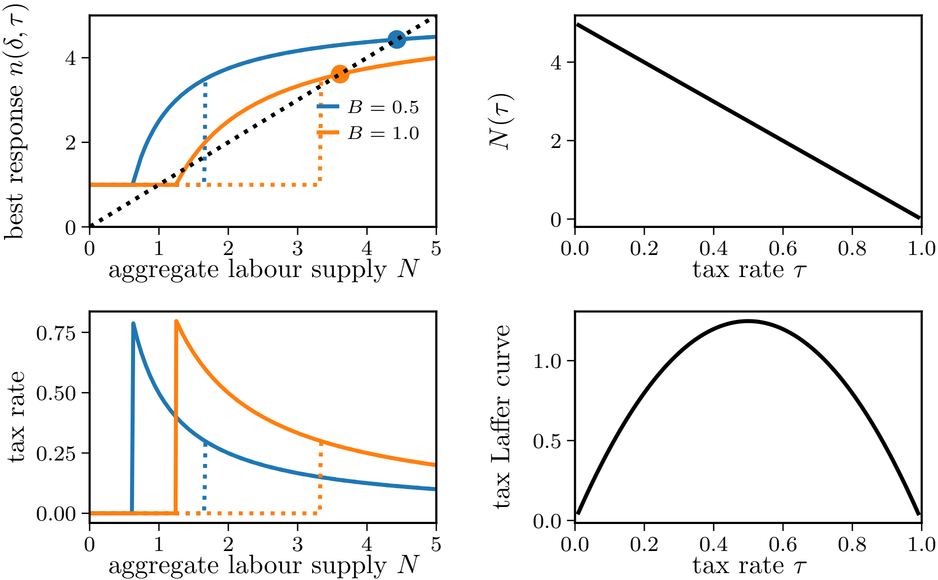
Yields

$$(\tau, \delta) = \begin{cases} \left(\frac{B}{N+q}, 1\right) & \text{if} \quad (a_1^{1/\rho} + a_2^{1/\rho}\mathbb{1}_{[\delta\geq 1]})^{\rho}(1-\tau)^{1-\rho} \geq a_1 \\ (0, 0) & \text{otherwise} \end{cases}$$

Ramsey outcome implementability depends on B



More details



Takeaways

With complementarities, multiple equilibria are possible

Even without time inconsistency, govt policy implementation requires strategies

Scope for tradeoff between optimality and uniqueness

References

Bassetto, Marco, "Equilibrium and government commitment," *Journal of Economic Theory*, September 2005, *124* (1), 79–105.

Calvo, Guillermo A, "Servicing the Public Debt: The Role of Expectations," *American Economic Review*, September 1988, *78* (4), 647–61.